On the use of Lagrangian Coherent Structures in direct assimilation of ocean tracer images

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Objectives of the study

Phytoplankton bloom
Malvinas currents
December 6, 2006
(Courtesy: NASA)

- The main objective of this study is to show that we can exploit **ocean tracer images** in **direct image assimilation schemes**
- We realize a **numerical experiment** using a **high resolution double-gyre idealized model of the North Atlantic Ocean** ($1/54°$).
- We will focus on:
  - Surface velocity fields
  - **Sea Surface Temperature (SST)**
  - mixed layer **phytoplankton (PHY)**
- We construct an **observation operator** based on the computation of **Lagrangian Coherent Structures**
- We study the **sensibility of a cost function** associated with this operator wrt the amplitude of a surface velocity perturbation (state variable)

Outline

Direct Image Assimilation

General concept

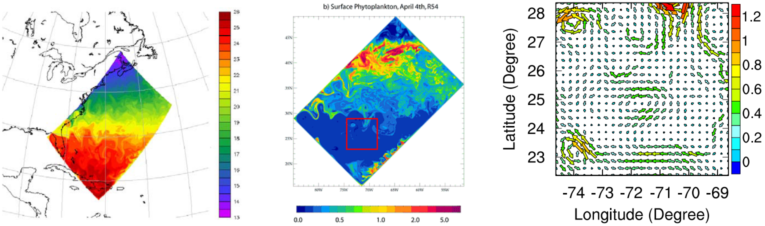
- **Ocean tracer images** contain structured information that should be exploited
- $\mathcal{S}$: **space of pertinent information** to be observed : **structures**
  - Frequency characteristics (e.g. multi-scale modelling of the images)
  - Pattern properties (contours, regions of interest . . .)
- $\|\cdot\|_{\mathcal{S}}$: **discrepancy measure** between two elements of $\mathcal{S}$
- $\mathcal{H}_{\mathcal{S}}$: structures **observation operators** (model equivalent of obs structures)

$$J(X_0) = \underbrace{\frac{1}{2}\int_0^T \|\mathcal{H}[\mathbf{X}] - \mathbf{y}_{obs}\|^2_{\mathcal{O}}\, dt}_{\text{classical term}} + \underbrace{\frac{1}{2}\int_0^T \|\mathcal{H}_{\mathcal{S}}[\mathbf{X}] - \mathbf{y}_s\|^2_{\mathcal{S}}\, dt}_{\text{"image" term}} + \frac{1}{2}\|x_0 - x_b\|^2_{\mathcal{X}}$$

- $\mathbf{y} \in \mathcal{S}$: observed structures in images (**sub-sampling** of observations)

Titaud *et al.*, 2010

Test case



(M. Lévy *et. al.*, 2009)

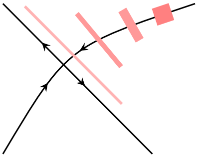
- High resolution (1/54°) idealized simulation of the North Atlantic Ocean (double gyre)
- NEMO-OPA/TOP2 (dynamics/tracers) and LOBSTER (bio-geochemical)
- **Sea Surface Temperature (SST)** and mixed layer **phytoplankton (PHY)**
- **Region of study**: $\Omega = [-74.62, -68.62] \times [22.36, 28.36]$ $(6^\circ \times 6^\circ)$
- **Reference date** : April 9

Sequence of **meso-scale surface velocities** (1/4°) obtained by sub-sampling and spatial filtering (Lanczos)

Coherent Lagrangian Structures (LCS)

The transport of a tracer in a fluid is closely related to emergent patterns called **Coherent Structures** (Ottino 1989, Wiggins 1992):

- Stationary flows: **stable and unstable manifolds of hyperbolic trajectories**
- Delimit regions of **whirls**, **stretching** or **contraction**



Stretching of a passive tracer in the vicinity of an hyperbolic point

- **In practice, LCS are determined by computing the Finite Time Lyapunov Exponents (FTLE)**

  (Haller and Yuan, 2000), (Haller, 2001a; 2001b; 2002; 2011), (Shadden *et al.*, 2005)
- **This tool is widely used in oceanography to study mixing processes**

  (d'Ovidio *et al.*, 2004), (Lehahn *et al.*, 2007), (Beron-Verra *et al.*, 2010)

Finite-Time Lyapunov Vectors (FTLV)

$$(\star)\begin{cases} \dfrac{D\mathbf{x}(t)}{Dt} = \mathbf{u}(\mathbf{x}(t), t) \\ \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}$$

Particle transport by the flow $\mathbf{u}(\mathbf{x}, t)$

$$\begin{cases} \dfrac{D\delta\mathbf{x}(t)}{Dt} = \nabla\mathbf{u}(\mathbf{x}(t), t).\delta\mathbf{x}(t) \\ \delta\mathbf{x}(t_0) = \delta_0, \quad \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}$$

Evolution of a given perturbation $\delta\mathbf{x}$

Finite-Time Lyapunov Vector

FTLV is defined as the direction of maximum stretching, i.e. **the eigenvector $\varphi_{t_0}^{t_0+T}(\mathbf{x}_0)$ of the largest eigenvalue $\lambda_{\max}$ of the Cauchy-Green strain tensor**:

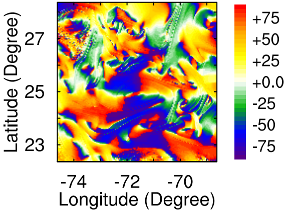
$$\Delta = \left[\nabla\phi_{t_0}^{t_0+T}(\mathbf{x}_0)\right]^* \left[\nabla\phi_{t_0}^{t_0+T}(\mathbf{x}_0)\right], \quad \phi_{t_0}^{t_0+T} : \mathbf{x_0} \mapsto \mathbf{x}(T), \quad \text{flow map of } (\star)$$

- **Backward FTLV** ($\approx$ stable manifold): time integration is inverted in $(\star)$
- Finite-Time Lyapunov Exponent (separation rate) : $\frac{1}{|T|} \ln \sqrt{\lambda_{\max}(\Delta)}$

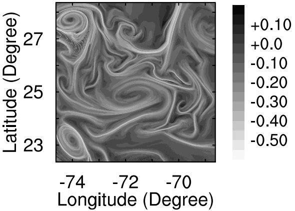
(Ott, 1993), (Shadden *et al.* ,2005; 2009), (Haller, 2011)

FTLV: variational point of view

- **FTLV is a local notion**: the eigenvector $\varphi_{t_0}^{t_0+T}$ is computed at a given point $\mathbf{x}_0$
- Seeding a domain with particles initially located on a grid leads to the computation of a discretized vector field



Backward FTLV orientation (Degree)



Backward FTLE (Day^{-1})

Backward integration
Meso-scale velocity field
Resolution $1/54°$

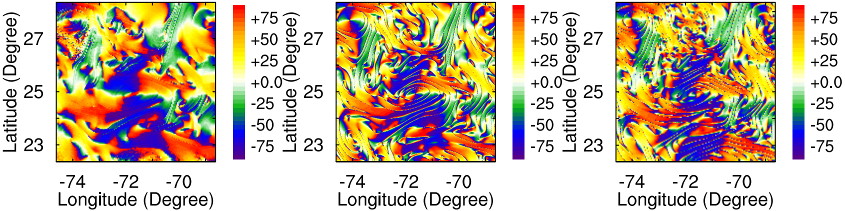
Backward FTLV orientation map with respect to the velocity field u

$$\Phi[\mathbf{u}] \;:\; \mathbf{x} \in \Omega \to \varphi_{t_0}^{t_0+T}(\mathbf{x}) \in \mathbb{R}^2$$

Connection between FTLV and tracer fields

The orientation of the gradient of passive tracers converge to that of backward FTLV in freely decaying 2D turbulence flow

(Lapeyre, 2002)



Backward FTLV orientation

∇ SST orientation

∇ PHY orientation

This property has also been observed on real data

(d'Ovidio *et al.*, 2009)

Observation Operator based on FTLV

- **Structure Space**: functions with values in the Euclidean sphere S^2

$$\mathcal{S} = \{f : \Omega \to S^2\}$$

- **Observation Operator** (vector field)

$$\mathcal{H}_{\mathcal{S}}(X) = \Phi(\mathbf{u}) \qquad \Phi(\mathbf{u}) \ : \ \mathbf{x} \in \Omega \mapsto \varphi_0^{-T}(\mathbf{x}) \in S^2$$

- **Information extraction** from the observed image c (vector field)

$$\mathrm{V} \ : \ \mathcal{I}_\Omega \to \mathcal{S} \qquad \mathrm{V}(c)(i,j) = \frac{\nabla c(i,j)}{\|\nabla c(i,j)\|} = \mathbf{y} \in S^2$$

- Orientation of $\mathbf{v} = (u,v) \in S^2$: $\Theta(\mathbf{v}) = \mathrm{atan}(v) \in [-\pi/2, \pi/2]$
- **Angular measure** in $\mathcal{S}$

$$\|f - g\|_{\mathcal{S}} = \sqrt{\frac{1}{n \times m} \sum_{i,j} \sin^2[\Theta(f(i,j)) - \Theta(g(i,j))]}$$

Corresponding image part of the cost function

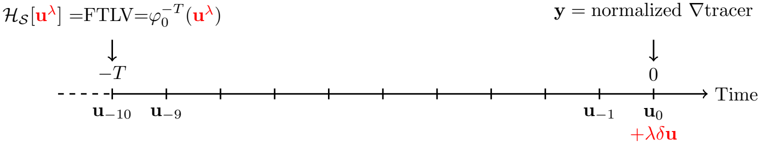
$$J_{\mathcal{S}}(\mathbf{u}) = \|\Phi(\mathbf{u}) - \mathrm{V}(c)\|^2_{\mathcal{S}}.$$

Methodology : Pre-requisite for data assimilation

Aim: study the behaviour of the cost function with respect to the **amplitude** λ **of velocity perturbations** on the form $\mathbf{u}_0 + \lambda\delta\mathbf{u}$, where $\delta\mathbf{u} \sim \mathcal{N}(0, \mathbf{SS}^{\mathrm{T}})$

We perturb the initial field of the sequence $(\mathbf{u}_k)_{k=-10}^{k=0}$:

$$\mathbf{u}_k^\lambda = \begin{cases} \mathbf{u}_0 + \lambda\delta\mathbf{u} & \text{if} \quad k = 0 \\ \mathbf{u}_k & \text{else} \end{cases} \qquad \mathbf{u}^\lambda = (\mathbf{u}_k^\lambda)_{k=-10}^{k=0}$$



Sensitivity of the cost function wrt to a perturbation amplitude λ:

$$\tilde{J}_\mathcal{S}(\lambda) = \|\mathcal{H}_\mathcal{S}[\mathbf{u}^\lambda] - \mathbf{y}\|_\mathcal{S}^2, \quad \lambda \in \Lambda.$$

- We have to check that the **sensitivity function** $\tilde{\mathcal{J}}_\mathcal{S}$ **admits a minimum at** $\lambda = 0$ (**no perturbation**).

Methodology

Climatological covariance matrix for the velocity perturbation

- $(\mathbf{u}^{(l)})_{l=1}^{r}$: first $r = 100$ EOFs of the one year sequence of simulated surface velocity fields

$$\mathbf{u}_k = \overline{\mathbf{u}} + \sum_{l=1}^{m=209} \alpha_k^{(l)} \mathbf{u}^{(l)},$$

- $\mathbf{S} = (\mathbf{u}^{(1)}|\mathbf{u}^{(2)}|\cdots|\mathbf{u}^{(r)})$: reduced rank square root representation of the climatological covariance matrix

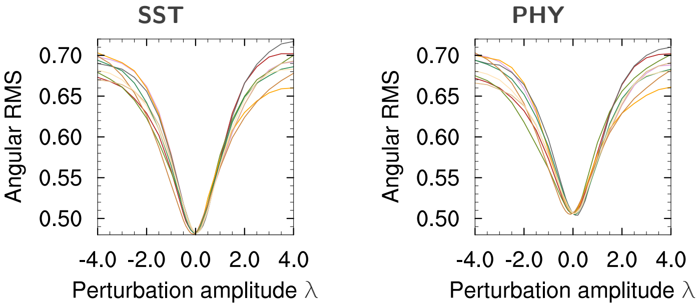
$$\mathbf{P} = \frac{1}{m}\sum_{k=1}^{m+1}(\mathbf{u}_k - \overline{\mathbf{u}})(\mathbf{u}_k - \overline{\mathbf{u}})^*$$

- **Gaussian perturbations with zero mean and covariance $\mathbf{S}\mathbf{S}^{\mathrm{T}}$**

$$\delta_{\mathbf{u}} \sim \mathcal{N}(0, \mathbf{S}\mathbf{S}^{\mathrm{T}}). \quad \delta_{\mathbf{u}} = \sum_{l=1}^{r} \mathbf{u}^{(l)} \delta x_l \quad \text{with} \quad \delta x_l \sim \mathcal{N}(0,1)$$

We are interested in perturbations of amplitude λ applied at the **reference date**:
$\mathbf{u}_0 + \lambda\delta\mathbf{u}$

Results / discussion



Variation of the sensitivity functions computed wrt the amplitude λ of nine random perturbations

- Each of the sensitivity function admits a **global minimum**
- **Minimum is generally reached around $\lambda = 0$ (no perturbations)**
- **Convex shape**: good point for minimization algorithms
- Minimum value is not zero

Conclusions, future work and references

Conclusions

- **High resolution** ocean tracer images may be exploited by a direct image assimilation scheme in a **mesoscale** model
- **FTLV fields contain information about the system dynamic** that can be observed in the ocean tracer fields: this is a good candidate to construct **observation operator** for image assimilation
- A single ocean tracer image contains a **time integrated information** on the system dynamics

Future work

- Full data assimilation experiment
- Observation errors / real data

References

- L. Gaultier, J. Verron, J.-M. Brankart, O. Titaud, P. Brasseur, On the inversion of submesoscale tracer fields to estimate the surface ocean circulation, *Journal of Marine Systems*, in press
- O. Titaud, J.-M. Brankart, J. Verron, On the use of Finite-Time Lyapunov Exponents and Vectors for direct assimilation of tracer images into ocean models, *Tellus A*, Oct. 2011
- O. Titaud, A. Vidard, I. Souopgui, and F.-X. Le Dimet. Assimilation of image sequences in numerical models. Tellus A, 62(1):30-47, Janvier 2010