Multi-Resolution Variational Method for Ocean Current Estimation from SST and Altimetry Obervations

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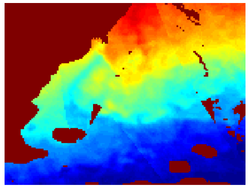
Geophysical variables

- **Sea surface temperature (SST)**
  - REMSS: low spatial resolution, low missing data rate
  - AVHRR-METOP: high spatial resolution, high missing data rate

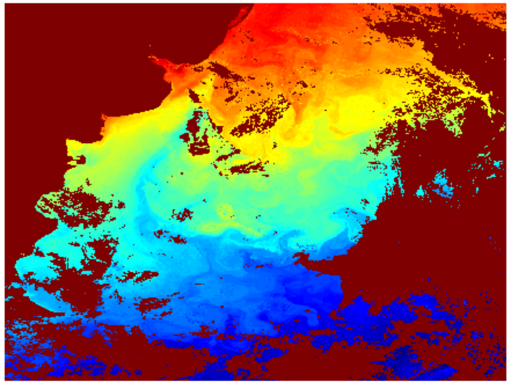
- **Chlorophyll concentration (CHL)**
  - MERIS: spatial resolution, high missing data rate

- **Altimetry**
  - Low spatio-temporalresolution

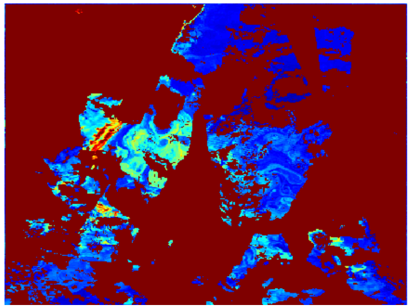
  (1 observation/week)



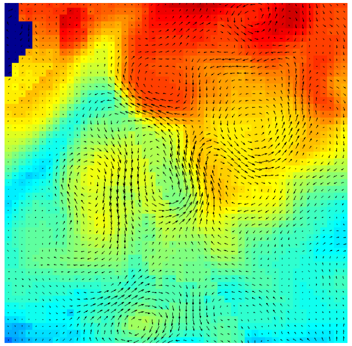
SST-AMSRE
25km



SST-METOP
5km



CHL-MERIS
15km



ALTIMETRY
30km

Problem: missing data interpolation

- **Formal problem:**

Hidden state (complete geophysical map)

$$Y_t = P_t X_t + \eta_t$$

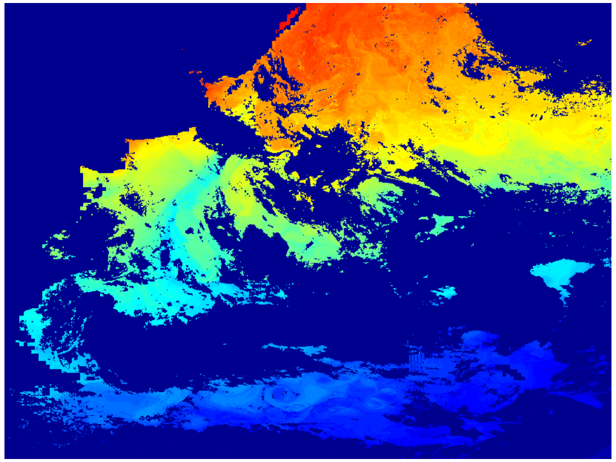
Projection operator

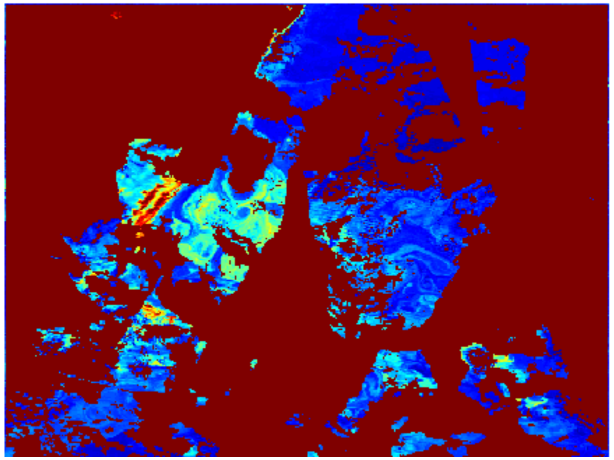
Gaussian noise
~N(0,R)

observation with missing data

- **Objective:**
  - recover the full geophysical maps
    - SST and altimetry

- **Our solution:**
  - Multi-resolution multi-modal variational data assimilation





Variational estimation : general principle

- **Data:**
  - Observation sequence Y_t, t Є [t_0 t_f] of geophysical variable X_t
  - Dynamical model $M(X_t)$: temporal evolution of X_t

- **Variational cost function**

$$J(X)=\mathcal{C}(X,Y)+\mathcal{C}(X)$$

- **C(X,Y): observation model**
  - similarity between reconstructed variables and observations

- **C(X): temporal consistency term**
  - Consistency of variables temporal evolution
  - Build on top of the dynamical model $M(X_t)$

Variational model

State model:

$$X = (\theta, \omega_g, u_a)$$

θ SST

ω_g geostrophic vorticity

u_a ageostrophic velocity

Dynamic model:

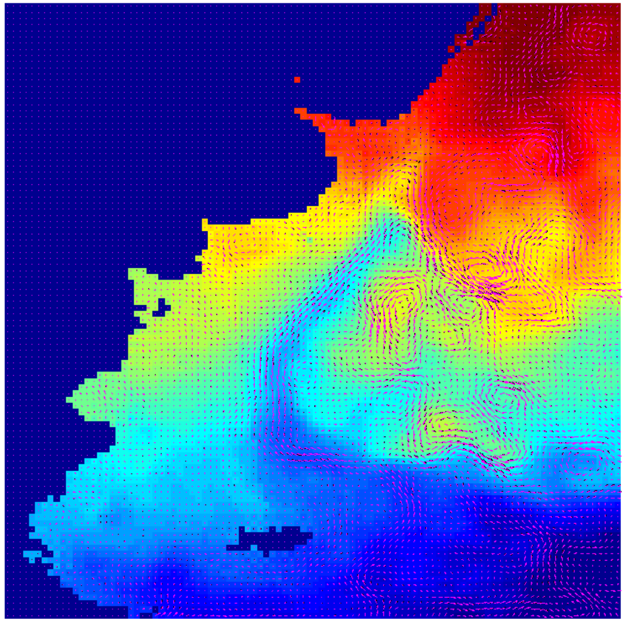
$$\begin{cases} \partial_t \theta + (u_a + u(\omega_g)) \nabla \theta + \kappa \mathrm{div}(\nabla \theta) \\ \partial_t \omega_g + u(\omega_g) \nabla \omega_g = \zeta_t \\ X_{t_0} = X_0 + \vartheta_t \end{cases}$$

Variational Problem:

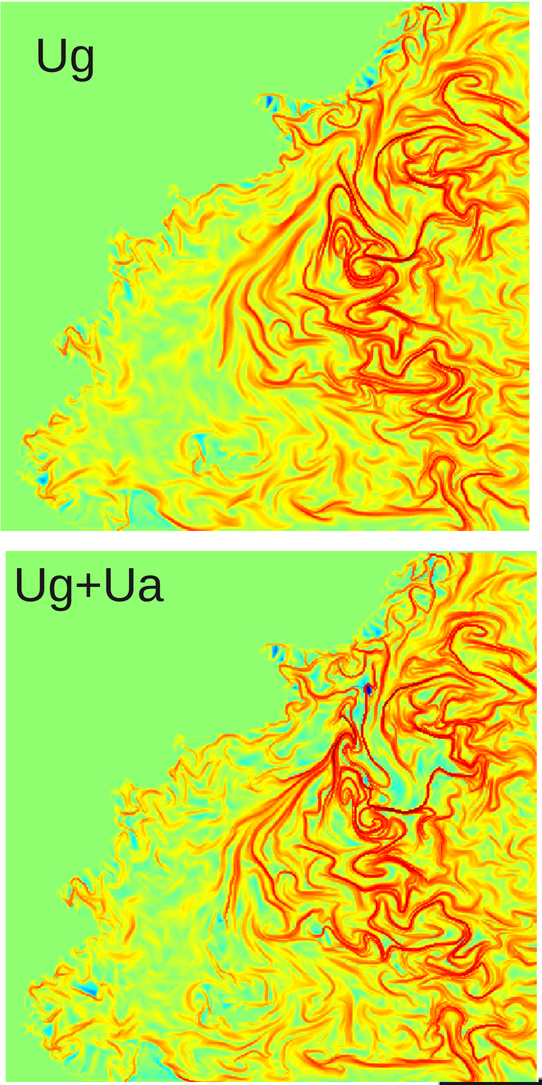
$$J(\theta, \omega_g, u_a) = \underbrace{E_{Obs}(\theta, \omega_g)}_{\text{observation term}} + \underbrace{E_{Dynamic}(\theta, \omega_g, u_a) + E_{Dynamic}(\omega_g)}_{\text{temporal consistency}} + \underbrace{E_{MultiResol}(\theta_{HR}, \theta_{LR})}$$

Results: circulation estimation

Black arrow=Ua+Ug
Magenta=Ug



Lyapunov exponents



- **Convergence structures are better resolved with the geostrophic and the ageostrophic velocities**

Thanks for your attention

Questions & Commentaires

Single (SST) resolution variational method

- **Hidden state sequence :**
- **Temporal evolution driven by the system**

$X = (\theta, \omega_g, u_a)$

θ	SST
ω_g	geostrophic vorticity
u_a	ageostrophic velocity

$$\begin{cases} \partial_t \theta + M(\theta, \omega_g, u_a) = \xi_t, \quad M(\theta, \omega_g, u_a) = (u_a + u(\omega_g)) \nabla \theta + \kappa \, \mathrm{div}(\nabla \theta) \\ \partial_t \omega_g + u(\omega_g) \nabla \omega_g = \zeta_t \\ X_{t_0} = X_0 + \vartheta_t \end{cases}$$

- **Variational problem**

$$J(\theta, \omega_g, u_a) = \underbrace{\int_{t_0}^{t_f} E(\theta_t, \omega_g) dt}_{\text{observation term}} + \underbrace{\int_{t_0}^{t_f} \left\| \partial_t \theta - M(\theta, \omega_g, u_a) \right\|_{Q_\theta^{-1}}^2 dt + \int_{t_0}^{t_f} \left\| \partial_t \omega_g - u(\omega_g) \nabla \omega_g \right\|_{Q_\omega^{-1}}^2}_{\text{temporal consistency}}$$

$$E(\theta, \omega_g) = \sigma_\theta^{-1} \left\| Y^\theta - P_\theta \theta \right\|_2^2 + \beta \left\| \nabla \theta \right\|_2^2 + \sigma_\omega^{-1} \left\| Y^\omega - P_\omega \omega_g \right\|_2^2$$

SST observation

vorticity observation (altimetry)

Multi-resolution variatonal method

- **Similar underlying dynamics**
  - Various geophysical variables follow the same ocean circulation
    - Same main frontal structures
  - Examples:
    - SST observation at various résolution (REMSS and METOP SST)
- **Solution:**
  - Multi-resolution variational processing with frontal structure contraints
- **Multi-resolution observation model:**

Regularity term: impose reconstructed variables to share main frontal structures

high resolution

low resolution

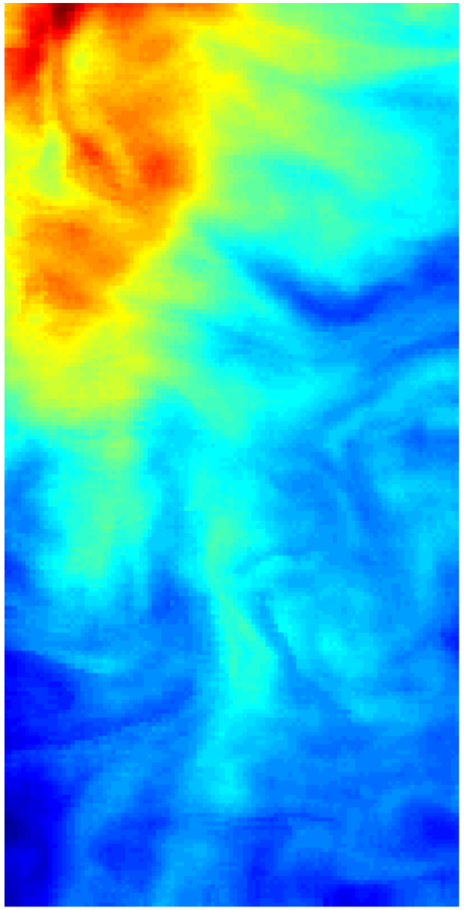
$$E_J(\theta_1,\theta_2)=E(\theta_1)+E(\theta_2)+\overbrace{\int_\Omega g_a(|\nabla\theta_1|)\rho_\varepsilon\left(<\frac{\nabla\theta_2}{|\nabla\theta_2|},\frac{\nabla\Lambda\theta_1^\perp}{|\nabla\Lambda\theta_1|}>\right)dp}$$

$$E(\theta)=\sigma^{-1}\left\|Y^\theta-P_\theta\theta\right\|_2^2+\beta\left\|\nabla\theta\right\|_2^2$$ Independant cost

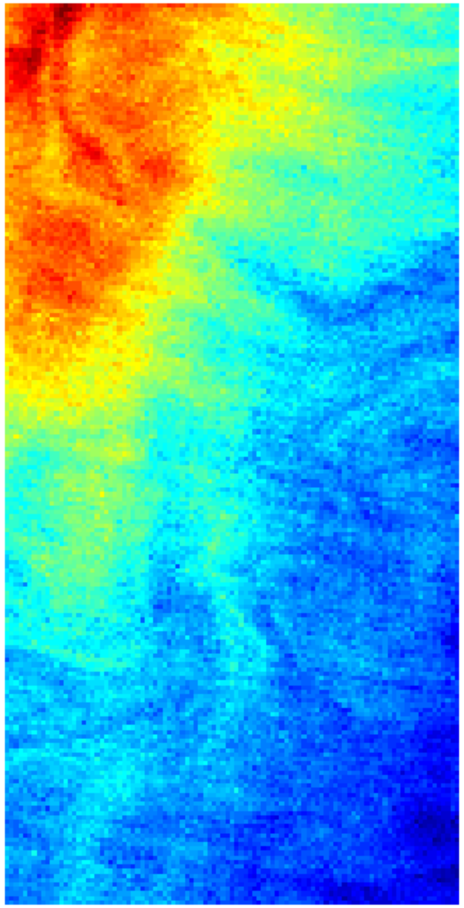
$$g_a(x)=1-\frac{1}{1+ax}$$ Frontal structure marker (edge detector)

$$\rho_\varepsilon(x)=\sqrt{x^2+\varepsilon}$$ Absolute value difrentiable approximation

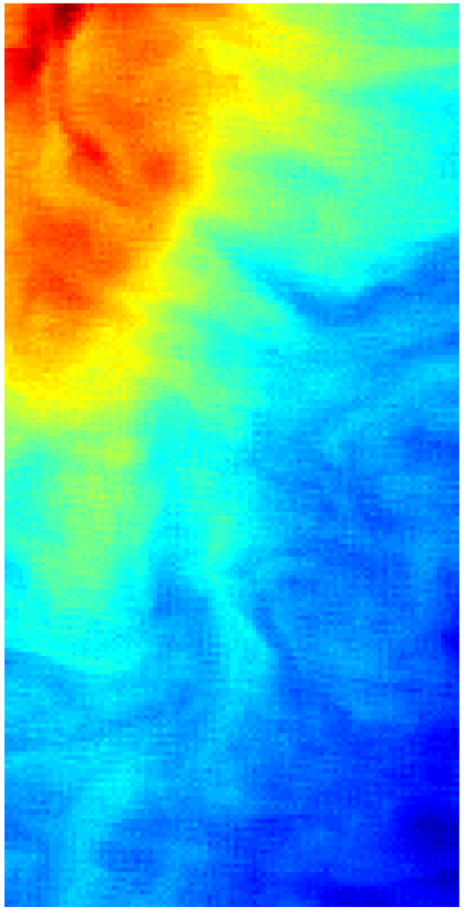
Results: frontal structure prior effects



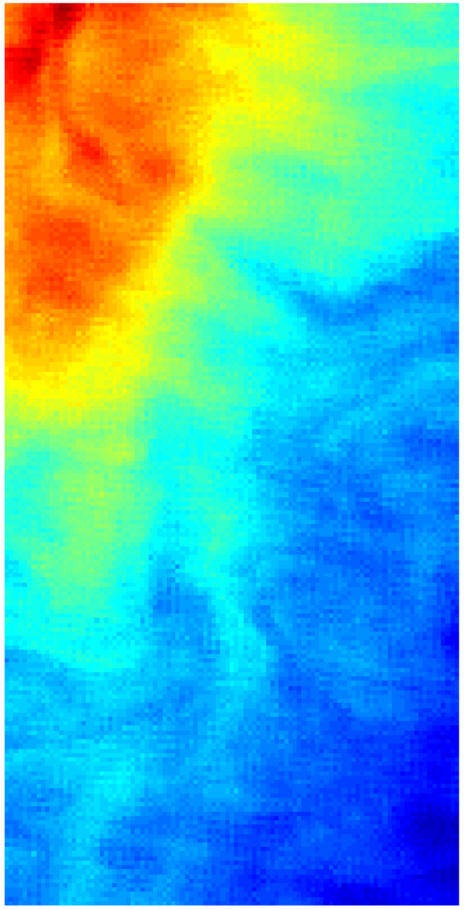
BT



SST



Regularized SST
BT fronts Prior



Regularized SST
BT div. Prior